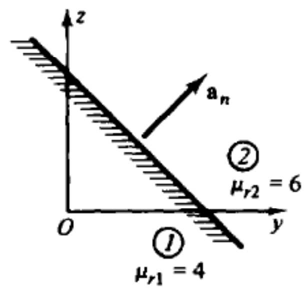

Fig. 13-5

Choosing the unit normal $\mathbf{a}_n = (\mathbf{a}_y + \mathbf{a}_z)/\sqrt{2}$,

$$B_{n1} = \frac{(2.0\mathbf{a}_x + 1.0\mathbf{a}_y) \cdot (\mathbf{a}_y + \mathbf{a}_z)}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\mathbf{B}_{n1} = \left(\frac{1}{\sqrt{2}}\right)\mathbf{a}_n = 0.5\mathbf{a}_y + 0.5\mathbf{a}_z = \mathbf{B}_{n2}$$

$$\mathbf{B}_{t1} = \mathbf{B}_1 - \mathbf{B}_{n1} = 2.0\mathbf{a}_x + 0.5\mathbf{a}_y - 0.5\mathbf{a}_z$$

$$\mathbf{H}_{t1} = \frac{1}{\mu_0}(0.5\mathbf{a}_x + 0.125\mathbf{a}_y - 0.125\mathbf{a}_z) = \mathbf{H}_{t2}$$

$$\mathbf{B}_{t2} = \mu_0\mu_{r2}\mathbf{H}_{t2} = 3.0\mathbf{a}_x + 0.75\mathbf{a}_y - 0.75\mathbf{a}_z$$

Now the normal and tangential parts of $\mathbf{B}_2$ are combined.

$$\mathbf{B}_2 = 3.0\mathbf{a}_x + 1.25\mathbf{a}_y - 0.25\mathbf{a}_z \quad \text{(T)}$$

$$\mathbf{H}_2 = \frac{1}{\mu_0}(0.50\mathbf{a}_x + 0.21\mathbf{a}_y - 0.04\mathbf{a}_z) \quad \text{(A/m)}$$

13.4. In region *1*, defined by $z < 0$, $\mu_{r1} = 3$ and

$$\mathbf{H}_1 = \frac{1}{\mu_0}(0.2\mathbf{a}_x + 0.5\mathbf{a}_y + 1.0\mathbf{a}_z) \quad \text{(A/m)}$$

Find $\mathbf{H}_2$ if it is known that $\theta_2 = 45°$.

$$\cos\alpha_1 = \frac{\mathbf{H}_1 \cdot \mathbf{a}_z}{|\mathbf{H}_1|} = 0.88 \qquad \text{or} \qquad \alpha_1 = 28.3°$$

Then, $\theta_1 = 61.7°$ and

$$\frac{\tan 61.7°}{\tan 45°} = \frac{\mu_{r2}}{3} \qquad \text{or} \qquad \mu_{r2} = 5.57$$

From the continuity of normal **B**, $\mu_{r1}H_{z1} = \mu_{r2}H_{z2}$, and so

$$\mathbf{H}_2 = \frac{1}{\mu_0}\left(0.2\mathbf{a}_x + 0.5\mathbf{a}_y + \frac{\mu_{r1}}{\mu_{r2}}1.0\mathbf{a}_z\right) = \frac{1}{\mu_0}(0.2\mathbf{a}_x + 0.5\mathbf{a}_y + 0.54\mathbf{a}_z) \quad \text{(A/m)}$$

13.5. A current sheet, $\mathbf{K} = 6.5\mathbf{a}_z$ A/m, at $x = 0$ separates region *1*, $x < 0$, where $\mathbf{H}_1 = 10\mathbf{a}_y$ A/m and region *2*, $x > 0$. Find $\mathbf{H}_2$ at $x = +0$.

Nothing is said about the permeabilities of the two regions; however, since $\mathbf{H}_1$ is entirely tangential, a change in permeability would have no effect. Since $B_{n1} = 0$, $B_{n2} = 0$ and therefore $H_{n2} = 0$.

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}$$
$$(10\mathbf{a}_y - H_{y2}\mathbf{a}_y) \times \mathbf{a}_x = 6.5\mathbf{a}_z$$
$$(10 - H_{y2})(-\mathbf{a}_z) = 6.5\mathbf{a}_z$$
$$H_{y2} = 16.5 \text{ (A/m)}$$

Thus, $\mathbf{H}_2 = 16.5\mathbf{a}_y$ (A/m).

13.6. A current sheet, $\mathbf{K} = 9.0\mathbf{a}_y$ A/m, is located at $z = 0$, the interface between region *1*, $z < 0$, with $\mu_{r1} = 4$, and region *2*, $z > 0$, $\mu_{r2} = 3$. Given that $\mathbf{H}_2 = 14.5\mathbf{a}_x + 8.0\mathbf{a}_z$ (A/m), find $\mathbf{H}_1$.

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The current sheet shown in Fig. 13-6 is first examined alone.

$$\mathbf{H}'_1 = \tfrac{1}{2}(9.0)\mathbf{a}_y \times (-\mathbf{a}_z) = 4.5(-\mathbf{a}_x)$$
$$\mathbf{H}'_2 = \tfrac{1}{2}(9.0)\mathbf{a}_y \times \mathbf{a}_z = 4.5\mathbf{a}_x$$

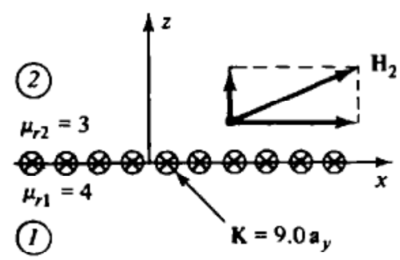

Fig. 13-6

From region *1* to region *2*, H_x will increase by 9.0 A/m due to the current sheet.

Now the complete **H** and **B** fields are examined.

$$\mathbf{H}_2 = 14.5\mathbf{a}_x + 8.0\mathbf{a}_z \quad \text{(A/m)}$$
$$\mathbf{B}_2 = \mu_0(43.5\mathbf{a}_x + 24.0\mathbf{a}_z) \quad \text{(T)}$$
$$\mathbf{B}_1 = \mu_0(22.0\mathbf{a}_x + 24.0\mathbf{a}_z) \quad \text{(T)}$$
$$\mathbf{H}_1 = 5.5\mathbf{a}_x + 6.0\mathbf{a}_z \quad \text{(A/m)}$$

Note that H_{x1} must be 9.0 A/m less than H_{x2} because of the current sheet. B_{x1} is obtained as $\mu_0\mu_{r1}H_{x1}$.

An alternate method is to apply $(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}$:

$$(H_{x1}\mathbf{a}_x + H_{y1}\mathbf{a}_y + H_{z1}\mathbf{a}_z) \times \mathbf{a}_z = \mathbf{K} + (14.5\mathbf{a}_x + 8.0\mathbf{a}_z) \times \mathbf{a}_z$$
$$-H_{x1}\mathbf{a}_y + H_{y1}\mathbf{a}_x = -5.5\mathbf{a}_y$$

from which $H_{x1} = 5.5$ A/m and $H_{y1} = 0$. This method deals exclusively with tangential **H**; any normal component must be determined by the previous methods.

13.7. Region *1*, $z < 0$, has $\mu_{r1} = 1.5$, while region *2*, $z > 0$, has $\mu_{r2} = 5$. Near $(0, 0, 0)$,

$$\mathbf{B}_1 = 2.40\mathbf{a}_x + 10.0\mathbf{a}_z \quad \text{(T)} \qquad \mathbf{B}_2 = 25.75\mathbf{a}_x - 17.7\mathbf{a}_y + 10.0\mathbf{a}_z \quad \text{(T)}$$

If the interface carries a sheet current, what is its density at the origin?

Near the origin,

$$\mathbf{H}_1 = \frac{1}{\mu_0\mu_{r1}}\mathbf{B}_1 = \frac{1}{\mu_0}(1.60\mathbf{a}_x + 6.67\mathbf{a}_z) \quad \text{(A/m)}$$
$$\mathbf{H}_2 = \frac{1}{\mu_0}(5.15\mathbf{a}_x - 3.54\mathbf{a}_y + 2.0\mathbf{a}_z) \quad \text{(A/m)}$$